3 TIME VALUE OF MONEY

Time value of money refers to the fact that a money in pocket today is worth more than a money promised at some time in the future. One reason for this is that one could earn interest while waited, so a money today would grow to more later. The trade-off between money now and money later thus depends on, among other things, the rate you can earn by investing.

3.1 Future value

Future value (FV) refers to the amount of money an investment will grow to over some period of time at some given interest rate. Put another way, future value is the cash value of an investment at some time in the future. Let us start by considering a single-period investment.

3.1.1 Investing for a Single Period

Suppose you invest $1 in a savings account that pays 100 percent interest per year. How much will you have in one year? You will have $2. This $2 is equal to your original principal of $1 plus $1 in interest that you earn. We say that $2 is the future value of $1 invested for one year at 100 percent, and we simply mean that $1 today is worth $2 in one year, given that 100 percent is the interest rate.

In general, if you invest for one period at an interest rate of $r$, your investment will grow to $(1 + r)$ per dollar invested. In our example, $r$ is 100 percent, so your investment grows to $1 +
1 = 2 dollars per dollar invested. You invested $1 in this case, so you ended up with $1*(1+1)= $2.

### 3.1.2 Investing for More Than One Period

Going back to our $1 investment, what will you have after two years, assuming the interest rate doesn’t change? If you leave the entire $2 in the bank, you will earn $2* 1 = 2 in interest during the second year, so you will have a total of $2* (1+1)= $4. This $4 is the future value of $1 in two years at 100 percent. Another way of looking at it is that one year from now you are effectively investing $2 at 100 percent for a year. This is a single-period problem, so you’ll end up with $2 for every dollar invested, or $2 * (1+1) = $4 total.

This $4 has four parts. The first part is the $1 original principal. The second part is the $1 in interest you earned in the first year, and the third part is another $1 you earn in the second year, for a total of $3. The last $1 you end up with (the fourth part) is interest you earn in the second year on the interest paid in the first year.

This process of leaving your money and any accumulated interest in an investment for more than one period, thereby reinvesting the interest, is called compounding. Compounding the interest means earning interest on interest, so we call the result compound interest. With simple interest, the interest is not reinvested, so interest is earned each period only on the original principal.

We now take a closer look at how we calculated the $4 future value. We multiplied $2 by 2 to get $4. The $2, however, was $1 also multiplied by 2. In other words:

$$4 = 2*2= (1*2)*2= 1*(2*2)= 1 * 2^2$$

Now let’s ask: How much would our $1 grow to after three years? Once again, in two years, we’ll be investing $4 for one period at 100 percent. We’ll end up with $2 for every dollar we invest, or $4 *2 $8 total. This $8 is thus:

$$8 = 4*2= (2*2)*2= ((1*2)*2) *2= 1*(2*2 *2)= 1 * 2^3$$

You’re probably noticing a pattern to these calculations, so we can now go ahead
and state the general result. As our examples suggest, the future value of $1 invested for $t$ periods at a rate of $r$ per period is:

$$\text{Future value} = \$1 \times (1 + r)^t$$

The expression $(1 + r)^t$ is sometimes called the *future value interest factor* (or just *future value factor*) for $1$ invested at $r$ percent for $t$ periods and can be abbreviated as $\text{FVIF}(r, t)$.

Let us consider what would your $1$ be worth after five years with interest rate 10%? We can first compute the relevant future value factor as:

$$(1 + r)^t = (1 + 0.10)^5 = 1.1^5 = 1.6105$$

Your $1$ will thus grow to:

$$\$1 \times 1.6105 = \$1.6105$$

### 3.2 Present value and discounting

When we discuss future value, we are thinking of questions like, What will my $1000 investment grow to if it earns a 10% percent return every year for the next six years? The answer to this question is what we call the future value of $1000 invested at 10 percent for six years (verify that the answer is $1610.51$).

There is another type of question that comes up even more often in financial management that is obviously related to future value. Suppose you need to have $10000 in 5 years, and you can earn 10 percent on your money. How much do you have to invest today to reach your goal?
3.2.1 The Single-Period Case

We now ask how much do we have to invest today at 10 percent to get $1 in one year? In other words, we know the future value here is $1, but what is the present value (PV)? The answer isn’t too hard to figure out. Whatever we invest today will be 1.1 times bigger at the end of the year. Because we need $1 at the end of the year:

Present value * 1.1 = $1

Or, solving for the present value:

Present value = $1/1.1 = $.909

In this case, the present value is the answer to the following question: What amount, invested today, will grow to $1 in one year if the interest rate is 10 percent? Present value is thus just the reverse of future value. Instead of compounding the money forward into the future, we discount it back to the present.

From our examples, the present value of $1 to be received in one period is generally given as:

\[ PV = \frac{1}{1 + r} \]

3.2.2 Present Values for Multiple Periods

Suppose you need to have $100 in two years. If you can earn 10 percent, how much do you have to invest to make sure that you have the $100 when you need it? In other words, what is the present value of $100 in two years if the relevant rate is 10 percent? Based on your knowledge of future values, you know the amount invested must grow to $100 over the two years. In other words, it must be the case that:

\[ $100 = \frac{PV}{1.1} / 1.1 = \frac{PV}{1.1^2} = \frac{PV}{1.21} \]

Given this, we can solve for the present value:
Present value = $100/1.121 = $82,644.6

Therefore, $82,644.6 is the amount you must invest in order to achieve your goal. As you have probably recognized by now, calculating present values is quite similar to calculating future values, and the general result looks much the same. The present value of $1 to be received t periods into the future at a discount rate of r is:

\[ PV = \frac{1}{(1 + r)^t} \]

The quantity in brackets, \( 1/(1 + r)^t \), goes by several different names. Because it’s used to discount a future cash flow, it is often called a discount factor. With this name, it is not surprising that the rate used in the calculation is often called the discount rate. We will tend to call it this in talking about present values. The quantity in brackets is also called the present value interest factor (or just present value factor) for $1 at r percent for t periods and is sometimes abbreviated as PVIF(r, t). Finally, calculating the present value of a future cash flow to determine its worth today is commonly called discounted cash flow (DCF) valuation.

### 3.3 Present versus Future Value

If you look back at the expressions we came up with for present and future values, you will see there is a very simple relationship between the two. We explore this relationship and some related issues in this section.

What we called the present value factor is just the reciprocal of (that is, 1 divided by) the future value factor:

\[ \text{Future value factor} = (1 + r)^t \]

\[ \text{Present value factor} = 1/(1 + r)^t \]

In fact, the easy way to calculate a present value factor on many calculators is to first...
calculate the future value factor and then press the “1/x” key to flip it over.
If we let $FV_t$ stand for the future value after $t$ periods, then the relationship between
future value and present value can be written very simply as one of the following:

$$\text{PV} \times (1 + r)^t = FV_t$$

$$\text{PV} = \frac{FV_t}{(1 + r)^t} = FV_t \times \frac{1}{(1 + r)^t}$$

This last result we will call the basic present value equation. We will use it throughout
the text. There are a number of variations that come up, but this simple equation underlies
many of the most important ideas in corporate finance.