5  RISK AND PORTFOLIO

First and basic thing we should know a risk is that risk occurs than and only than when we expect that something in the future. The risk is the possibility of difference between our expectations and what really happened. If you don’t care about future, you have no risk. It means each state of future has the same utility. Risk is everywhere and concerns everybody, because everybody expects something, even if he or she is not conscious about this.

We know the discount rate for safe projects, and we have an estimate of the rate for average-risk projects. But we don’t know yet how to estimate discount rates for assets that do not fit these simple cases. To do that, you have to learn (1) how to measure risk and (2) the relationship between risks borne and risk premiums demanded.

5.1 Rates of return

First, we assume no inflation in economy so all the rates are nominal. If investor buy an asset of any sort, one’s gain (or loss) from that investment is called the return on investment. This return usually have two components. First, receive some cash directly while one’s own the investment. This is called the income component of your return. Second, is the change in value of the purchased asset and it’s called capital gain (or loss) on investment. However, the term is also used to mean percentage return, which is e.g. stock's total return - dividend plus change in value - divided by the investment amount (Figure 2).
Suppose investor bought the stock at the beginning of 2010 when its price was $10.00 a share. By the end of the year the value of that investment had appreciated to $15.00 giving a capital gain of

\[ \$15 - \$10 = \$5. \]

In addition, in 2010 the company paid a dividend of $1 a share.

The return on investment is therefore:

\[
\text{Nominal return} = \text{capital gain} + \text{dividend} = (15 - 10) + 1 = 6
\]

Percentage rate of return is income on an investment expressed as a percentage of the investment's purchase price. With a common stock, the rate of return is dividend yield, or your annual dividend divided by the price you paid for the stock, so the percentage return on investment was:

\[
\text{Percentage return} = \frac{\text{capital gain} + \text{dividend}}{\text{initial share price}} = \frac{(15 - 10) + 1}{10} = \frac{6}{10} = 60\
\]

The percentage return can also be expressed as the sum of the dividend yield and percentage capital gain. The dividend yield is the dividend expressed as a percentage of the stock price at the beginning of the year:

\[
\text{Dividend yield} = \frac{\text{dividend}}{\text{initial share price}} = \frac{1}{10} = 10\%
\]
Similarly, the percentage capital gain is

\[
\text{Percentage capital gain} = \frac{\text{capital gain}}{\text{initial share price}} = \frac{15 - 10}{10} = \frac{5}{10} = 50\%
\]

Thus the total return is the sum of 10% + 50% = 60%.

### 5.2 Variance and Standard Deviation

While there may be different definitions of risk, one widely used measure is variance. Variance measures the variability of realized returns around an average level. The larger the variance the higher the risk in the portfolio. The standard statistical measures of volatility are variance and standard deviation.

The variance of the market return is the expected average value of squared deviations from mean. squared deviation from the expected return. In other words,

\[
\text{Variance } \sigma^2 = \text{expected value of } (\bar{r} - r)^2
\]

where \( \bar{r} \) is the actual return and \( r \) is the expected return.

The standard deviation is often used by investors to measure the risk of a stock or a stock portfolio. The basic idea is that the standard deviation is a measure of spread: the more a stock's returns vary from the stock's average return, the more volatile the stock.

The standard deviation is simply the square root of the variance:

\[
\text{Standard deviation } \sigma = \sqrt{\text{expected value of } (\bar{r} - r)^2}
\]

Consider the following two stock portfolios and their respective returns (in per cent) over the last six months. Both portfolios end up increasing in value from $1,000 to $1,058. However,
they clearly differ in volatility. Portfolio A's monthly returns range from -1.5% to 3% whereas Portfolio B's range from -9% to 12%.

The standard deviation of the returns is a better measure of volatility than the range because it takes all the values into account. The standard deviation of the six returns for Portfolio A is 1.52; for Portfolio B it is 7.24.

Here is a another very simple example showing how variance and standard deviation are calculated. Suppose that you are offered the chance to play the following financial game. You start by investing $1000. Your return depends on combination of two different states of economy:

- Changing interest rates by FED
- Changing interest rates by EBC

For each interest decreasing that comes up you get back your starting balance plus 10 percent, and for each interest increasing that comes up you get back your starting balance less 5 percent. Clearly there are four equally likely outcomes:

Possible states of economy:
* FED-decreasing, EBC-decreasing: You gain 10+10=20 percent.
* FED-decreasing, EBC-increasing: You gain 10-5=5 percent.
* FED-increasing, EBC-decreasing: You gain -5+10=5 percent.
* FED-increasing, EBC-increasing: You lose -5+-5 =-10 percent.

There is a chance of 1 in 4, or .25, that you will make 20 percent; a chance of 2 in 4, or .5, that you will make 5 percent; and a chance of 1 in 4, or .25, that you will lose 10 percent. The game’s expected return is, therefore, a weighted average of the possible outcomes:

<table>
<thead>
<tr>
<th>FED</th>
<th>EBC</th>
<th>Return</th>
<th>Probability</th>
<th>av-return</th>
<th>(av-return)^2</th>
<th>(av-re)^2*p</th>
</tr>
</thead>
<tbody>
<tr>
<td>decresing</td>
<td>decresing</td>
<td>20%</td>
<td>25%</td>
<td>5.00%</td>
<td>-15%</td>
<td>0.0225</td>
</tr>
<tr>
<td>decresing</td>
<td>increasing</td>
<td>5%</td>
<td>25%</td>
<td>1.25%</td>
<td>0%</td>
<td>0.0000</td>
</tr>
<tr>
<td>increasing</td>
<td>decresing</td>
<td>5%</td>
<td>25%</td>
<td>1.25%</td>
<td>0%</td>
<td>0.0000</td>
</tr>
<tr>
<td>increasing</td>
<td>increasing</td>
<td>-10%</td>
<td>25%</td>
<td>-2.50%</td>
<td>15%</td>
<td>0.0225</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td></td>
<td></td>
<td></td>
<td>0.0113</td>
</tr>
</tbody>
</table>

Table 2.
Table 2 shows that the variance of the percentage returns is 0,0113 Standard deviation is the square root of variance. This figure is in the same units as the rate of return, so we can say that the game’s variability is 10,61 percent. 

One way of defining uncertainty is to say that more things can happen than will happen. The risk of an asset can be completely expressed, as we did for the coin tossing game, by writing all possible outcomes and the probability of each. In practice this is cumbersome and often impossible. Therefore we use variance or standard deviation to summarize the spread of possible outcomes. These measures are natural indexes of risk. If the outcome of the game had been certain, the standard deviation would have been zero. The actual standard deviation is positive because we don’t know what will happen. Or think of a second game, again, there are four equally likely outcomes:

Table 3

<table>
<thead>
<tr>
<th>FED</th>
<th>EBC</th>
<th>Return</th>
<th>Probability</th>
<th>av-return</th>
<th>(av-return)^2</th>
<th>(av-re)^2*p</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreasing</td>
<td>decreasing</td>
<td>40%</td>
<td>25%</td>
<td>10%</td>
<td>-35%</td>
<td>0,1225</td>
</tr>
<tr>
<td>decreasing</td>
<td>increasing</td>
<td>5%</td>
<td>25%</td>
<td>1%</td>
<td>0%</td>
<td>0,0000</td>
</tr>
<tr>
<td>increasing</td>
<td>decreasing</td>
<td>5%</td>
<td>25%</td>
<td>1%</td>
<td>0%</td>
<td>0,0000</td>
</tr>
<tr>
<td>increasing</td>
<td>increasing</td>
<td>-30%</td>
<td>25%</td>
<td>-8%</td>
<td>35%</td>
<td>0,1225</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5%</td>
<td></td>
<td>0,0613</td>
</tr>
</tbody>
</table>

For this game the expected return is 10 percent, the same as that of the first game. But its standard deviation is more than double that of the first game 24,75 versus 10,61 percent. By his measure the second game is more than twice as risky as the first\(^1\).

\(^1\) Estimating future returns using historical data investors means adaptive expectations (not rational) assumption. If everybody acts this way he creates positive spillovers in system. In short time we have exponential growth, but then always unexpectedly comes big decline. So the proper name for modern portfolio theory should be modern bubble creating theory.
5.3 *Diversification and Risk Reducing*

We have already concentrated on individual assets considered separately, but most investors hold a portfolio of assets. Investors tend to own more than just a single stock, bond, or other asset. Given that this is so, portfolio return and portfolio risk are of obvious relevance. Accordingly, we now discuss portfolio expected returns and variances.

5.3.1 Portfolio Weights

There are many equivalent ways of describing a portfolio. The most convenient approach is to list the percentage of the total portfolio’s value that is invested in each portfolio asset. We call these percentages the portfolio weights.

Table 4

<table>
<thead>
<tr>
<th>Asset</th>
<th>USD</th>
<th>portfolio weights</th>
<th>portfolio weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset A</td>
<td>$200,00</td>
<td>(Asset A)/(Asset A+Asset B+Asset C)</td>
<td>20%</td>
</tr>
<tr>
<td>Asset B</td>
<td>$300,00</td>
<td>(Asset B)/(Asset A+Asset B+Asset C)</td>
<td>30%</td>
</tr>
<tr>
<td>Asset C</td>
<td>$500,00</td>
<td>(Asset C)/(Asset A+Asset B+Asset C)</td>
<td>50%</td>
</tr>
<tr>
<td>Total</td>
<td>$1,000,00</td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4 shows simple example. Investor has $200 in asset A, $300 in asset B and $500 in asset C then total portfolio is worth $1000. The percentage of the portfolio is summarize in the last column: the asset A is $200/$1000, or 20%, the asset B is $300/$1000, or 30%, the asset C is $500/$1000, or 50%. The weights of the portfolio are thus 0.20, 0.30 and 0.50.
easy to see that the weights have to add up to 100% because all investor’s amount is invested somewhere.

5.3.2 Portfolio Expected Returns

Let’s go now consider the portfolio of the same three assets, but the portfolio weights are different: Asset 40%, Asset B and Asset C 30%. Rate of return depends on two different state of economy, each with the same probability 50%. First in Table 5 we calculate expected returns, variance and standard deviations for each asset.

Table 5

<table>
<thead>
<tr>
<th>FED</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Asset C</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreasing</td>
<td>20%</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>increasing</td>
<td>-10%</td>
<td>0%</td>
<td>-10%</td>
</tr>
<tr>
<td>Expected return</td>
<td>5,00%</td>
<td>15,00%</td>
<td>0,00%</td>
</tr>
<tr>
<td>var</td>
<td>0,0450</td>
<td>0,0450</td>
<td>0,0200</td>
</tr>
<tr>
<td>st dev</td>
<td>21,21%</td>
<td>21,21%</td>
<td>14,14%</td>
</tr>
<tr>
<td>portfolio weights</td>
<td>40%</td>
<td>30%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Now we calculate the rates of expected return on this portfolio. To answer these questions, suppose the FED increased interest rates and economy actually declines. In this case, 40 percent of invested money loses 10 percent, 30 percent of invested money does not change their value and the other 30 percent also loses 10 percent gains 30 percent. The portfolio return, $r_p$, in a depression is thus:

$$r_p = 0.40 \times (-10\%) + 0.30 \times 0 + 0.30 \times (-10\%) = -7\%$$
Table 6 summarizes the remaining calculations. When FED decreases interests, during boom the portfolio will return 20 percent:

$$r_p = 0.40 \times 20\% + 0.30 \times 30\% + 0.30 \times 10\% = 20\%$$

Table 6

<table>
<thead>
<tr>
<th>FED</th>
<th>Return</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Asset C</th>
<th>Probability</th>
<th>av-return</th>
<th>av-return</th>
</tr>
</thead>
<tbody>
<tr>
<td>decresing</td>
<td>20%</td>
<td>30%</td>
<td>10%</td>
<td>50%</td>
<td>20,00%</td>
<td>10,0%</td>
<td></td>
</tr>
<tr>
<td>incresing</td>
<td>-10%</td>
<td>0%</td>
<td>-10%</td>
<td>50%</td>
<td>-7,00%</td>
<td>-3,5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6,50%</td>
<td></td>
</tr>
</tbody>
</table>

As indicated in Table 6, the expected return on portfolio, $E(r_p)$, is 6.5 percent. We can save ourselves some work by calculating the expected return more directly. Given these portfolio weights, we could have reasoned that we expect 40 percent of our money to earn 5 percent, 30 percent to earn 15 percent and 30 percent to earn 0 percent. Our portfolio expected return is thus:

$$E(r_p) = 0.40 \times E(r_A) + 0.30 \times E(r_B) + 0.30 \times E(r_C) =$$

$$0.40 \times 5\% + 0.30 \times 15\% + 0.30 \times 0\% = 6.5\%$$

This is the same portfolio expected return we calculated previously. This method of calculating the expected return on a portfolio works no matter how many assets there are in the portfolio. Suppose we had $n$ assets in our portfolio, where $n$ is any number. If we let $w_i$ stand for the percentage of our money in Asset $i$, then the expected return would be:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$
This says that the expected return on a portfolio is a straightforward combination of the expected returns on the assets in that portfolio. This seems somewhat obvious, but, as we will examine next, the obvious approach is not always the right one.

5.3.3 Portfolio Variance

Now we must find the risk of portfolio and to understand the effect of diversification, we need to know how the risk of a portfolio depends on the risk of the individual shares. We now, that expected return on a portfolio that contains investment in Assets A, B and C is 6.5 percent. At first one may be inclined to assume that the standard deviation of your portfolio is a weighted average of the standard deviations of the two stocks. Simple intuition suggest that by analogy the risk of portfolio, measured by standard deviation is (Table 7):

\[
\sigma_p = \sigma_A p_A + \sigma_B p_B + \sigma_C p_C = 0.40 \times 21.21\% + 0.30 \times 21.21\% + 0.30 \times 14.14\% = 19.09\% .
\]

Unfortunately, this approach is completely wrong as a rule, That would be correct only if the prices of the two stocks moved in perfect lockstep, so this is just special extremely rare case with which we will deal later.

| Table 7 |
|------------------|------------------|------------------|------------------|------------------|
| FED Return       | Asset A          | Asset B          | Asset C          | Portfolio        |
| Expected         | 5.00%            | 15.00%           | 0.00%            | 6.50%            |
| return           | Var              | 0.0450           | 0.0450           | 0.0200           | 0.04             |
| st dev           | 21.21%           | 21.21%           | 14.14%           | 19.09%           |
| portfolio weights| 40%              | 30%              | 30%              | Wrong!           |
Let’s see what the standard deviation really is. Table 8 summarizes the relevant calculations. As we see, the portfolio’s variance is about .018, and its standard deviation is less than we thought - it’s only 13.5 percent. What is illustrated here is that the variance on a portfolio is not generally a simple combination of the variances of the assets in the portfolio.

First basic formula for standard deviation is:

\[ \sigma_p^2 = \sum_{i=1}^{n} [p_j (r_j - E(r_p))]^2 \]

\( p_j \) – probability of \( j \)-th state of economy

### Table 8

<table>
<thead>
<tr>
<th>FED</th>
<th>Return</th>
<th>Probability</th>
<th>av-return</th>
<th>(av-return)^2</th>
<th>(av-return)^2*p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset A</td>
<td>20%</td>
<td>50%</td>
<td>20,00%</td>
<td>0,018225</td>
<td>0,009113</td>
</tr>
<tr>
<td>Asset B</td>
<td>30%</td>
<td>-10%</td>
<td>-7,00%</td>
<td>0,018225</td>
<td>0,009113</td>
</tr>
<tr>
<td>Asset C</td>
<td>10%</td>
<td>50%</td>
<td>-3,5%</td>
<td>0,018225</td>
<td>0,009113</td>
</tr>
</tbody>
</table>

5.3.4 Calculating portfolio risk - correlation

As we see calculating the expected portfolio return is easy. The hard part is to work out the risk of the portfolio. Let us assume that in the past the standard deviation of returns was \( r_A \) percent for Asset A and \( r_B \) percent for Asset B. We believe that these figures are a good forecast of the spread of possible future outcomes. Now, the most important question is how strong and in which way this two assets are “connected”

Variance of portfolio is dependent on the way in which individual securities interact with each other. To obtain the portfolio variance of a two-stock we can use such formula:
The portfolio standard deviation is, of course, the square root of the variance.

$$\sigma_p^2 = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \sigma_{AB} = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{AB} \sigma_A \sigma_B$$

Let’s have a closer look at this formula. So first, we must weigh the variance of the returns on Asset A by the square of the proportion invested in it. Similarly, we weight the variance of the returns on Asset B by the square of the proportion invested in Asset B. The first two parts of this formula depend on the variances of Asset A and 2 and the last part of this formula depend on their covariance. Just as you weighted the variances by the square of the proportion invested, so you must weigh the covariance by the product of the two proportionate holdings $w_A$ and $w_B$.

The covariance essentially tells us whether or not two securities returns are correlated. The covariance can be expressed as the product of the correlation coefficient and the two standard deviations:

$$\sigma_{AB} = \text{cov}_{AB} = \rho_{AB} \sigma_A \sigma_B,$$

Covariance measures by themselves do not provide an indication of the degree of correlation between two securities. As such, covariance is standardized by dividing covariance by the product of the standard deviation of two individual securities. This standardized measure is called the correlation coefficient.

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$$

Correlation is the covariance of security A and B divided by the product of the standard deviation of these two securities. It is a pure measure of the co-movement between the two securities and is bounded by $-1$ and $+1$. For the most part stocks tend to move together. In this case the correlation coefficient is positive, and therefore the covariance is also positive.
Portfolio risk can be effectively diversified (reduced) by combining securities with returns that do not move in tandem with each other. A correlation of +1 means that the returns of the two securities always move in the same direction; they are perfectly positively correlated.

\[ \rho_{12} = 1 \]

\[ \sigma_p^2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 = (w_1 \sigma_1 + w_2 \sigma_2)^2 \]

\[ \sigma_p = w_1 \sigma_1 + w_2 \sigma_2 \]

So we have analogical result as \( r_p \).

In any other case, diversification reduces the risk below this figure.

\[ 0 < \rho_{12} < 1 \]

Positive correlation coefficient is typical in real economy and it means that we have limited possibility of risk reduction.

A correlation of –1 means the returns always moved in the opposite direction and are negatively correlated.

\[ \rho_{12} = -1 \]

\[ \sigma_p^2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 - 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 - 2 w_1 w_2 \sigma_1 \sigma_2 = (w_1 \sigma_1 - w_2 \sigma_2)^2 \]

\[ \sigma_p = w_1 \sigma_1 - w_2 \sigma_2 < w_1 \sigma_1 + w_2 \sigma_2 \]

If the stocks just tended to move in opposite directions, the correlation coefficient and the covariance would be negative.

\[ -1 < \rho_{12} < 0 \]

And the last interesting example when correlation is zero means the two securities are independent and have no relationship to each other.

If the prospects of the stocks were wholly unrelated, both the correlation coefficient and the covariance would be zero;
So, assets are independent, $\rho_{12}=0$, equally weighted, so $w_1 = w_2 = w_n$, equally risky, $\sigma_1 = \sigma_2 = \cdots$, with different rates of return

$$\sigma_p^2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2 = \sigma_n^2 w_n^2 + \sigma_n^2 w_n^2 = 2\sigma_n^2 w_n^2$$

$$\sigma_p^2 = n \sigma_n^2 \frac{1}{n^2} = \sigma_n^2 \frac{1}{n}$$

$$\sigma_p = \frac{\sigma_n}{\sqrt{n}}$$

$$\lim_{n \to \infty} \sigma_p = \lim_{n \to \infty} \frac{\sigma_n}{\sqrt{n}} = 0$$

We can see that in this case we can much reduce the risk by putting very large amount of assets to our portfolio.

In Table 9 we have an example of portfolio of two Assets.

<table>
<thead>
<tr>
<th>FED</th>
<th>Return</th>
<th>Probability</th>
<th>av-return</th>
<th>av-return</th>
<th>(av-return)^2</th>
<th>(av-return)^2*p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Asset A</td>
<td>Asset B</td>
<td>portfolio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>decreasing</td>
<td>24%</td>
<td>3,0%</td>
<td>50%</td>
<td>13,50%</td>
<td>6,8%</td>
<td>0,000506</td>
</tr>
<tr>
<td></td>
<td></td>
<td>increasing</td>
<td>12%</td>
<td>6,0%</td>
<td>9,00%</td>
<td>4,5%</td>
</tr>
</tbody>
</table>

$$\text{var portf} = 0,00051$$

$$\text{st portf} = 2,25\%$$

Expected return 18,00% 4,50% 12,25%

$$\text{var} = 0,0072$$ 0,0005

$$\text{st dev} = 8,49\%$$ 2,12%

| portfolio weights | 50% | 50% |
Table 10

<table>
<thead>
<tr>
<th>FED</th>
<th>Return</th>
<th>Probability</th>
<th>av-return</th>
<th>(av-return)^2</th>
<th>(av-re)^2*p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asset A</td>
<td>Asset B</td>
<td>portfolio</td>
<td>portfolio weight</td>
<td></td>
</tr>
<tr>
<td>decresing</td>
<td>24%</td>
<td>3,0%</td>
<td>50%</td>
<td>7,20%</td>
<td>3,6%</td>
</tr>
<tr>
<td>inincrezing</td>
<td>12%</td>
<td>6,0%</td>
<td>50%</td>
<td>7,20%</td>
<td>3,6%</td>
</tr>
<tr>
<td></td>
<td>7,2%</td>
<td>var portf=</td>
<td>st portf=</td>
<td>0,00%</td>
<td></td>
</tr>
<tr>
<td>Expected return</td>
<td>18,00%</td>
<td>4,50%</td>
<td>7,2%</td>
<td>var</td>
<td>0,0072</td>
</tr>
<tr>
<td></td>
<td>8,49%</td>
<td>2,12%</td>
<td>st dev</td>
<td>st portf=</td>
<td>0,00%</td>
</tr>
<tr>
<td></td>
<td>portfolio weights</td>
<td>20%</td>
<td>80%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3.5 Systematic and unsystematic risk. Risk diversification

There are important differences among various sources of risk. Some part of risk is directed specifically at particular companies, and some part of risk is more general. Unique risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors.

First, diversifiable risk (also known as unsystematic or specific risk) represents the portion of an asset’s risk and risk factors affecting only that firm. It’s attributable to firm-specific events, such as strikes, lawsuit, regulatory actions, and loss of a key account. Unsystematic risk is due to factors specific to an industry or a company like labor unions, product category, research and development, pricing, marketing strategy etc.

While the market risk (also known as systematic risk) risk is concerned with economy-wide (macroeconomic) factors which affect all firms. This is the relevant portion of an asset’s risk attributable to market factors that affect all firms such as war, inflation, international incidents, and political events. Market risk stems from the fact that there are other economy wide perils that threaten all businesses.
Unsystematic risk that is associated with random causes, can be eliminated through diversification. Company or industry specific risk that is inherent in each investment. The amount of unsystematic risk can be reduced through appropriate diversification. So, unsystematic risk is economy-wide sources of risk that affect the overall stock market. It cannot be eliminated through diversification and the combination of a security’s non-diversifiable risk and diversifiable risk is called total risk.

Systematic risk is due to risk factors that affect the entire market such as investment policy changes, foreign investment policy, change in taxation clauses, shift in socio-economic parameters, global security threats and measures etc. Systematic risk is beyond the control of investors and cannot be mitigated to a large extent. In contrast to this, the unsystematic risk can be mitigated through portfolio diversification. It is a risk that can be avoided and the market does not compensate for taking such risks.

Total Risk = Systematic Risk + Non-Systematic Risk.

Total risk is then the sum of these two. So, specific (or unsystematic) risk arises from factors which are random as between one firm’s securities and those of another. As these factors are random, a reasonably small amount of diversification will cause them to cancel one another.
Non-systematic risk is the risk that disappears in the portfolio construction process when you diversify among assets that are not correlated. Diversification works because prices of different stocks do not move exactly together.

If we select a number of marketable securities at random, form them into portfolios of varying sizes, measure the expected returns and standard deviation of returns from each of our various-sized portfolios and then plot standard deviation against size of portfolio, we should obtain a graph similar to that in Figure 4.

The market portfolio is made up of individual stocks, but its variability doesn’t reflect the average variability of its components. It means that diversification reduces variability. Even a little diversification can provide a substantial reduction in variability.

**Figure 4 Market risk and size of portfolio**

Systematic risk is the risk that remains after constructing the market portfolio, which presumably contains all risky assets. It is the risks that cannot be diversified away. The various-sized portfolios are constructed randomly. Several portfolios of each size are
constructed, and the standard deviation shown in Figure 4 is the average of the standard deviations of all of the portfolios for each size.

As standard deviation of returns around the mean is a reasonable measure of risk, Figure 4 suggests that large reduction in risk can be achieved merely by randomly combining securities in portfolios.

Two features might surprise. These are:

(a) Very limited diversification yields large reductions in the level of risk. Even spreading the investment funds available into a couple of different securities successfully eliminates a large amount of risk. Each additional different security added to the portfolio yields successively less by way of risk reduction.

(b) Once the portfolio contains about 15 to 20 securities there is little to be gained by way of risk reduction from further increases in its size. There seems to be part of the risk which is impervious to attempts to reduce it through diversification.

The implications of the phenomenon represented in Figure 4 are:

(a) investors should hold securities in portfolios as, by doing so, risk can be reduced at little cost; and

(b) there is limited point in diversifying into many more than 15 to 20 different securities as nearly all the benefits of diversification have been exhausted at that size of portfolio. Further diversification means that the investor will have to pay higher dealing charges to establish the portfolio and then will have more cost and/or work in managing it.

If systematic risk must be borne by investors because it is caused by economy-wide factors, it seems likely that some securities are more susceptible to these factors than are others. For example, a high street food supermarket would seem likely to suffer less than would a manufacturer of capital goods, say, as a result of economic recession.

Is the level of risk attaching to the returns from all securities the same, even though no one is forced to bear any specific risk? This question and several others relating to it will now be examined.

In Figure 4 we have divided the risk into its two parts—unique risk and market risk. If you have only a single stock, unique risk is very important; but once you have a portfolio of 20 or more stocks, diversification has done the bulk of its work. For a reasonably well-diversified portfolio, only market risk matters. Therefore, the predominant source of
uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor’s portfolio with it.

5.3.6 Limits of diversification

Some risk potentially can be eliminated by diversification, there is also some risk that you can’t avoid, regardless of how much you diversify. Generally market risk affects each asset and that is why stocks have a tendency to move together. And that is why investors are exposed to market uncertainties, no matter how many stocks they hold.

If the prospects of the stocks were wholly unrelated, both the correlation coefficient and the covariance would be zero;

All assets in portfolio are equally weighted, so $w_1 = w_2 = w_n = 1/n$, and $\sigma$ is average variance and $\sigma_{ij}$ is average covariance

First we start our calculations with portfolio of two assets:

$$\sigma_p^2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2w_1w_2\sigma_{12} = 2\left(\frac{1}{2}\right)^2 \bar{\sigma} + (2^2 - 2)\left(\frac{1}{2}\right)^2 \bar{\sigma}_{ij}$$

In case of n-asset portfolio:

$$\sigma_p^2 = n\left(\frac{1}{n}\right)^2 \bar{\sigma} + (n^2 - n)\left(\frac{1}{n}\right)^2 \bar{\sigma}_{ij} = \left(\frac{1}{n}\right)\sigma + \left(1 - \frac{1}{n}\right)\bar{\sigma}_{ij}$$

So with $n$ goes to infinity we have:

$$\sigma_p^2 = \lim_{n \to \infty} \left(\frac{1}{n}\right)\sigma + \left(1 - \frac{1}{n}\right)\bar{\sigma}_{ij} = \bar{\sigma}_{ij}$$
5.3.7 Beta

Market risk is measured by beta, which is another measure of investment risk that is based on the volatility of returns. In contrast to standard deviation, beta measures volatility relative to a relevant baseline rather than to the mean of the asset that is being evaluated. Beta is the appropriate measure of an asset's contribution to your portfolio's risk, as it measures only systematic risk, i.e., market risk.

You should recall that the standard form of the equation of the regression line is \( Y = a + bX \). When you regress one asset on another asset or a benchmark index, the slope of the regression line, \( b \), is referred to as the dependent variable's beta and it describes the movement of the asset (the dependent variable) relative to its benchmark (the independent variable).

Based on the monthly data, the growth fund's beta is 1.2, which tells us that for every 1% move in the S&P 500 the growth fund can be expected to move approximately 1.2% in the same direction. The market's beta is by definition 1.0 and it is the baseline market risk. The risk-free asset's beta is necessarily 0.0.

In this example, there is very little dispersion of the data points with respect to the regression line. This is to be expected, as the dependent variable, being a large-cap fund, should be very highly correlated with the S&P 500 and it is a well-diversified portfolio. If we did the same exercise with an individual large-cap stock, the data points would probably be much more dispersed. Also, the individual stock's market risk, beta, could be considerably higher or lower than 1.0, depending on the nature of the firm.

Beta is a commonly published statistic that you can use to evaluate the market risk of assets that you are considering adding to your portfolio. However, betas are usually derived using the S&P 500 as a baseline, which is fine if you're evaluating large-cap domestic stocks or if you want to see how any particular asset moves relative to the S&P 500. Just keep in mind that betas thus derived are relative to the S&P 500 and that they only represent the residual volatility.

Portfolio betas are calculated as the weighted average of the betas of the assets that comprise the portfolios. If your universe extends well beyond the S&P 500, to get a truly relevant
portfolio beta, the individual asset's betas would have to be derived using a baseline that is representative of the assets in the portfolio, i.e., the market risk of your universe is still 1.0 by definition but it may be more or less volatile than the S&P 500. For a broadly diversified portfolio, this would entail developing a weighted average index of the appropriate indexes. I'm not suggesting that you need to do this, I'm merely making the point. Portfolio standard deviation is all you need to estimate your portfolio's probable variability.

As beta is a measure of risk, it can be related to standard deviation. Indeed, an asset's beta is equal to the product of its correlation coefficient, \( R \), and its standard deviation divided by the market's standard deviation. Mathematically, the correlation coefficient divided by the market's standard deviation factors the specific risk out of the asset's standard deviation leaving only the systematic risk, which, as you now know, is the market risk of the asset.

We can summarize that:

if \( \beta = 0 \)

asset is risk free

if \( \beta = 1 \)

asset return = market return

if \( \beta > 1 \)

asset is riskier than market index

if \( \beta < 1 \)

asset is less risky than market index

An asset's systematic risk, relative to the average, can be measured by its beta coefficient, The risk premium on an asset is then given by its beta coefficient multiplied by the market risk premium, \([E(R_M) - R_f] * \beta\)
The expected return on an asset, \( E(R_i) \), is equal to the risk-free rate, \( R_f \), plus the risk premium:

\[ E(R_i) = R_f - [E(R_m) - R_f] \times \beta \]

So risk free rate is some kind of reference point adjusted by the risk which has qualitative and quantitative part.

This is the equation of the SML, and it is often called the capital asset pricing model (CAPM). This chapter completes our discussion of risk and return. Now that we have a better understanding of what determines a firm’s cost of capital for an investment, the next several chapters will examine more closely how firms raise the long-term capital needed for investment.